SEM-2 (MATHEMATICS GENERAL)

(ANUP RAY)

MIKADO
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Date

Solve the following problems:

1) $p^3 - 4xyp + 8y^2 = 0$ 2) $y = 2px + y^2p^3$

3) $x = py + ap^2$ 4) $y = 2px + p^2y$

c) Equations solvable for y

Let the equation (1) solvable for y. Then y can be expressed explicitly in terms of x and p, say $y = F(x, p)$ --- (2)
Differentiating (2) w.r.t. x we get

$p = \phi(x, p, \frac{dp}{dx})$ (say) --- (3), an equation containing two variables x and p.
Let the solution of the equation (3) be of the form $\psi(x, p, c) = 0$ --- (4) where c being an arbitrary constant.

Eliminating p from (2) & (4) we get the solution of the given equation (1). If the elimination of p is not possible, then expressing x, y in terms of p and c, the required general primitive, in the parametric form in terms of the parameter p, is obtained.

Ex: Solve $y = 3x + \log p$.

Ans: we have $y = 3x + \log p$ --- (1)
Differentiating (1) w.r.t. x we have

$$p = 3 + \frac{1}{p}\frac{dp}{dx} \quad \text{or} \quad dx = \frac{dp}{p(p-3)}$$

$$\text{or} \quad dx = \frac{1}{3}\left[\frac{1}{p-3} - \frac{1}{p}\right]dp$$

Integrating both sides we get,

$$x = \frac{1}{3}\log(p-3) - \frac{1}{3}\log p - \frac{1}{3}\log c,$$

c being an arbitrary constant.

$$\text{or } e^{3x} = \frac{p-3}{pc} \quad \text{or } p = \frac{3}{1-ce^{3x}}$$

Putting this value of p in (1), we get the required solution of (7) as

$$y = 3x + \log\left\{\frac{3}{1-ce^{3x}}\right\}$$

Ex: Solve $4y = x^2 + p^2$

Ans: Differentiating the given equation w.r.t x we get

$$4p = 2x + 2p\frac{dp}{dx} \quad \text{or } \frac{dp}{dx} = \frac{2p-x}{p}$$

which is a homogeneous equation. —(1)

Let $p = vx$ or $\frac{dp}{dx} = v + x\frac{dv}{dx}$, then (1) gives

$$v + x\frac{dv}{dx} = \frac{2v-1}{v} \quad \text{or } x\frac{dv}{dx} = -\frac{(v-1)^2}{v}$$

$$\text{or } \frac{dx}{x} = -\frac{v\,dv}{(v-1)^2} = -\left[\frac{(v-1)+1}{(v-1)^2}\right]dv$$

$$\text{or } \frac{dx}{x} = -\frac{dv}{v-1} - \frac{dv}{(v-1)^2}$$

Integrating we get,

$$\log x = -\log(v-1) + \frac{1}{v-1} + c,$$

c being an arbitrary constant.

$$\text{or } \log\{x(v-1)\} = \frac{1}{v-1} + c$$

$$\text{or } \log(p-x) = \frac{x}{p-x} + c \quad \text{--- (2)}$$

Given equation and eqn. (2) together give the solution, p being the parameter.